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1988 J. Phys. A: Math. Gen. 21 L559

(http://iopscience.iop.org/0305-4470/21/10/005)

J. Phys. A: Math. Gen. 21 (1988) L559–L562. Printed in the UK

LETTER TO THE EDITOR

Specific heat and crossover exponents at the tricritical collapse of trails

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Received 10 March 1988

Abstract. From the exact enumeration on a triangular lattice of trail configurations at the tricritical regime as the fugacity for intersections is increased, we find $\phi_t = 0.68\ (8)$ for the crossover exponent (and hence $\alpha_t = 0.5\ (2)$ for the specific heat exponent). It indicates the collapse transition of trails to differ from that of polymer chains at the θ point in two dimensions.

In a recent series of papers [1, 2] evidence was presented for a new tricritical point in trails [3] (intersecting but non-overlapping walks), as the fugacity for intersections is increased. From extensive exact enumerations for the weighted number of configurations and series of the end-to-end distance, first estimates for the tricritical exponents γ_t and ν_t were derived [1, 2]. We recall that this tricritical point separates the swollen regime in which the trails have the scaling properties of self-avoiding walks [3–5] from a collapsed compact phase in which the size exponent $\nu = 1/d$. A similar collapse was known for a very long time to occur at the θ point of a self-avoiding chain due to monomer–monomer attraction [6]. The Hamiltonian (magnetic analogue) for the trail-generating function [5] has a different symmetry at the tricritical point than that describing the θ point. In particular for the trail $d^* = 4$ is the upper critical dimension [5] while $d^* = 3$ is that for the θ point [6]. Thus a new universality class with non-Gaussian behaviour in three dimensions is expected for the collapse of trails. Unfortunately the perturbative renormalisation group fails to exhibit a tricritical fixed point in $d = 4 - \varepsilon$ dimensions [5]. Therefore other methods, such as series expansions, are necessary to study this non-perturbative tricritical point.

The evidence for this new tricritical point in three dimensions,, presented in [2], is quite convincing since the value obtained for the exponent of the number of configurations $\gamma_t = 0.43\ (5)$ practically rules out the possibility of a Gaussian behaviour with $\gamma_t = 1$.

In two dimensions the results for the exponents found are $\nu_t = 0.525\ (25)$, $\gamma_t = 1.25\ (2)$ for the square lattice [1] and $\nu_t = 0.52\ (1)$, $\gamma_t = 1.18\ (2)$ for the triangular lattice [1]. These values are too close to those conjectured as exact for the θ point [7], $\nu_\theta = 0.5774$ and $\gamma_\theta = 1.1428$, to rule out the possibility for the two tricritical points to share the same universality class.

We were therefore motivated to look for other exponents in order to clarify the issue of whether the tricritical trails are in the θ point universality class.

0305-4470/88/100559+04$02.50

In this letter we report first results for the crossover exponent $\phi = 0.68$ (8) and the specific heat exponent $\alpha = 2 - \phi^{-1} = 0.5$ (2). These values are drastically different from those of the θ point, $\alpha_\theta = -\frac{1}{3}$ and $\phi_\theta = \frac{3}{7}$ [7]. These results are based on an analysis of the specific heat of trails from their exact enumeration (up to $l = 15$) on the triangular lattice. The series of cubic lattices, which suffer from the interference of the 'antiferromagnetic' singularity, were too noisy to be analysed. In figure 1 we show the specific heat plots for the triangular lattice as presented in [1]. They represent the curves of $h_l(\theta)$ for $l = 9$–15, which are defined by

$$h_l(\theta) = \frac{1}{l}\frac{\mathrm{d}^2}{\mathrm{d}\theta^2} \ln C_l(\theta) = \langle I^2(\theta)\rangle_l - \langle I(\theta)\rangle_l^2 \tag{1}$$

where

$$C_l(\theta) = \sum_{I>0} n(l, I) \exp(I\theta) \tag{2}$$

and $n(l, I)$ are the total number of configurations with length l and I intersections. So $h_l(\theta)$ are the approximate specific heats to order l and represent the relative fluctuations in the number of intersections as θ (the chemical potential (the fugacity is $f = \exp\theta$) for intersection) is varied. Their divergence at θ_t is a signature of the collapse transition. In figure 2 we have plotted the maxima of each $h_l(\theta)$ (as θ is varied) against the order l. The points, corresponding to $l > 9$, seem to be well on the onset of an asymptotic behaviour of the form

$$\max h_l(\theta) \sim l^x. \tag{3}$$

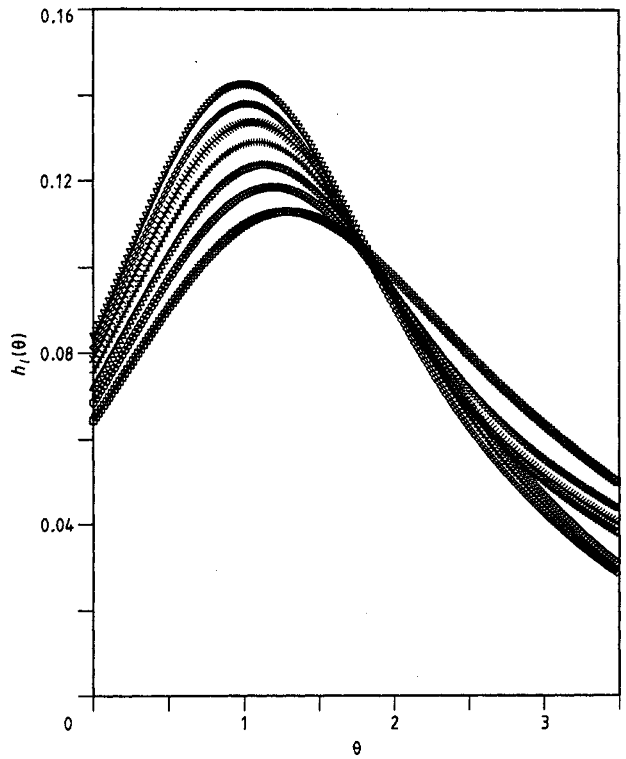

Figure 1. The specific heat plots $h_l(\theta)$ for $l = 9, \ldots, 15$.

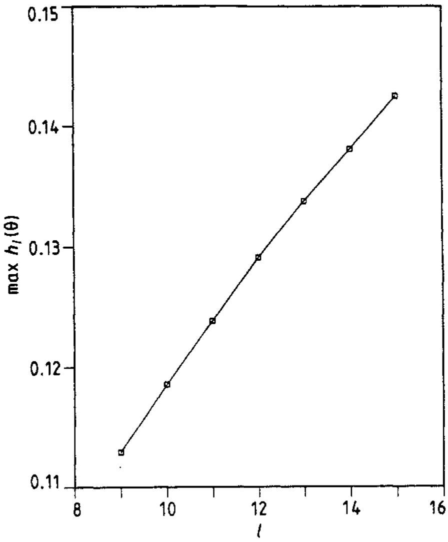

Figure 2. The plot of max $h_l(\theta)$ against l, for $l=9,\ldots,15$.

From the scaling behaviour of the specific heat

$$h_l(\theta) \sim (\theta-\theta_t)^{-\alpha} g[(\theta-\theta_t) l^{\phi}] \tag{4}$$

it follows that

$$x = \phi\alpha \tag{5}$$

where θ_t is the tricritical value of the coupling θ and g is a scaling function.

To derive an estimate for $x=\phi\alpha$ we had to avoid as much as possible the non-singular contributions which are relatively important at small l. We therefore proceeded as follows. For a given value of x we computed the slopes D_l between consecutive points in the curve at max $h_l(\theta)$ against l^x:

$$D_l = [\max h_l(\theta) - \max h_{l-1}(\theta)]/[l^x - (l-1)^x]. \tag{6}$$

We look to minimise the differences between the D_l and choose as optimal x the value which minimises the expression

$$\Delta(l_i, l_f) = \sum_{l=l_i}^{l_f} [D_l/D_{l-1} - 1]^2. \tag{7}$$

We repeated these calculations for different pairs (l_i, l_f) with $l_i = 11$, 12, 13 and $l_f = 13, 14, 15$. The values extracted for $\phi = \frac{1}{2}(1+x)$ with different pairs upon minimising $\Delta(l_i, l_f)$ are quoted in table 1. The values for ϕ quoted above follow from the minimal

Table 1. Values of $\phi = \frac{1}{2}(1+x)$ derived from minimising $\Delta(l_i, l_f)$ (equation (7)).

l_f	l_i		
	11	12	13
13	0.689	—	—
14	0.604	0.601	—
15	0.704	0.765	0.670

($\phi = 0.60$) and maximal ($\phi = 0.77$) values in table 1. The values of α were then extracted from the relation $\alpha = 2 - \phi^{-1}$.

These values represent an independent evidence that in two dimensions as well the trail tricritical point is not in the θ point universality class.

After completion of this work it was reported that MC calculations [8] yield values of ϕ and α in agreement with these quoted above.

This work was supported in part by the Xerox Webster Research Center and by the University of Rochester.

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